15116 3 Hours / 100 Marks

Seat No.

Instructions: (1) All Questions are *compulsory*.

- (2) Answer each next main Question on a new page.
- (3) Illustrate your answers with neat sketches wherever necessary.
- (4) Figures to the right indicate full marks.
- (5) Assume suitable data, if necessary.
- (6) Use of Non-Programmable Electronic Pocket Calculator is permissible.
- (7) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Attempt any TEN of the following:

20

(a) Find missing term, if
$$\begin{vmatrix} 4 & 3 & 9 \\ 3 & -2 & 7 \\ 11 & 4 & -- \end{vmatrix} = 0$$

(b) If
$$\begin{bmatrix} 3 & -6 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
, find a, b, c, d

(c) If
$$A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$$
, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$, find X such that $2X + 3A - 4B = I$.

(d) If
$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$
, $B = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$, find $A^T + B^T$ and $A^T - B^T$.

(e) Resolve into the partial fraction
$$\frac{1}{x^3 + 3x^2 + 2x}$$

- (f) Prove that $\cos A = \cos^2(A/2) \sin^2(A/2)$
- (g) Without using calculator find the value of sin 75°.
- (h) Without using calculator find the value of cos (3660).
- (i) Prove that $\sin (A + \pi/6) \sin(A \pi/6) = \cos A$

- (j) Prove that $\cos [\sin^{-1}(3/5)] = \frac{4}{5}$
- (k) State the condition of two lines are parallel and perpendicular to each other.
- $(l) \qquad \hbox{Calculate the range from the following data:} \\$

Weight in kg: 70, 75, 69, 80, 85, 83, 65, 89, 73, 84, 90

2. Attempt any FOUR of the following:

16

(a) Solve the following equations by using Cramer's Rule:

$$\frac{5}{x+2} + \frac{3}{y+1} = 2$$

$$\frac{10}{x+2} - \frac{3}{y+1} = 1$$

- (b) Find x, y, z if $\begin{bmatrix} 2+x & -1 & 3 \\ 0 & y & z \\ 4 & 1 & 3 \end{bmatrix} + \begin{bmatrix} 1+x & 2 & 3 \\ 0 & 1+y & 4 \\ 2 & 3 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 1 & 6 \\ 0 & -1 & 6 \\ 6 & 4 & 8 \end{bmatrix}$
- (c) If $A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ 3 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$, find $(AB) \cdot C$.
- (d) Find Inverse of Matrix, $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$
- (e) Resolve into partial fractions, $\frac{x^3 + 1}{x^2 + 2x}$.
- (f) Resolve into partial fractions, $\frac{2x+3}{x^2(x-1)}$

3. Attempt any FOUR of the following:

16

(a) Solve the following equations, by using matrix intersection method :

$$x + 3y + 2z = 6$$
, $3x - 2y + 5z = 5$, $2x - 3y + 6z = 7$

(b) Resolve into partial fractions $\frac{x}{x^3 + 1}$.

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- (c) Resolve into partial fractions, $\frac{e^{x} + 1}{(e^{x} + 2)(e^{x} + 3)}$
- (d) Prove that $\sin (\pi + \theta) = -\sin \theta$
- (e) Find value of $\frac{\sec^2 135^{\circ}}{\cos (-240^{\circ}) 2 \sin (930^{\circ})}$
- (f) Prove that $\cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\left(\frac{27}{11}\right)$

4. Attempt any FOUR of the following:

16

- (a) Prove that, $\cos (A + B) = \cos A \cos B \sin A \sin B$.
- (b) Prove that $\tan A \cdot \tan (60^{\circ} A) \cdot \tan (60^{\circ} + A) = \tan 3A$.
- (c) By using principal value, prove that

$$\sin^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 2\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) + 3\sin^{-1}(-1) = -\frac{\pi}{4}.$$

- (d) Prove that, (without using calculator) $\sin 20^{\circ} \cdot \sin 40^{\circ} \cdot \sin 60^{\circ} \cdot \sin 80^{\circ} = 3/16$
- (e) Prove that, $\cos C \cos D = -2 \sin \left(\frac{C+D}{2}\right) \sin \left(\frac{C-D}{2}\right)$
- (f) Prove that, $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \cos^{-1}\left(\frac{33}{65}\right)$

5. Attempt any FOUR of the following:

16

- (a) Prove that, $\frac{\sec 4\theta 1}{\sec 2\theta 1} = \frac{\tan 4\theta}{\tan \theta}$
- (b) Prove that, $\frac{\sin \theta \sin 5\theta + \sin 9\theta \sin 3\theta}{\cos \theta \cos 5\theta \cos 9\theta + \cos 13\theta} = \cot 4\theta.$
- (c) Prove that, $\sin^{-1} x = \cot^{-1} \left(\frac{\sqrt{1-x^2}}{x} \right)$.
- (d) Prove that distance between two parallel lines $ax + by + c_1 = 0$ and

$$ax + by + c_2 = 0$$
 is $\left| \frac{c_1 - c_2}{\sqrt{A^2 + B^2}} \right|$

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- (e) Find equation of lines passing through (12, -4) and whose sum of the intercepts is equal to 10.
- (f) If m_1 and m_2 are the slopes of the lines, then prove that the angle between the two lines is $\theta = \tan^{-1} \left| \frac{m_1 m_2}{1 + m_1 m_2} \right|$.

6. Attempt any FOUR of the following:

16

(a) Prove that the length of perpendicular from the point $p(x_1, y_1)$ to the line

$$Ax + By + C = 0 \text{ is } \left| \frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}} \right|$$

- (b) Find the length of the perpendicular from the point (2, 3) on the line 4x 6y 3 = 0.
- (c) Calculate the mean deviation from mean for the following data:

Marks: 3 4 5 6 7 8

No. of student: 1 3 7 5 2 2

(d) Find the standard deviation of the following:

Class: 0-20 20-40 40-60 60-80 80-100Frequency: 20 130 220 70 60

(e) Find variance from the following series:

Age under: 10 20 30 40 50 60 70 80 No. of person: 15 30 53 100 110 115 125 75

(f) The mean and variance of 5 items are 64 and 68 respectively. If two more items of values 62 and 66 are added to the data, find the new variance of 7 items.
